Research Article

APPROXIMATED AREA CALCULATION BY USING DISCRETE LEAST SQUARES METHOD

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Abstract: *This research delves into the application of the discrete least squares method for approximating the area of Pulau Ketam, an island located in Selangor, Malaysia. The study utilizes 24 data points collected from a detailed map of Pulau Ketam to demonstrate the effectiveness of the method. A user-friendly Graphical User Interface (GUI) named AreaXpert is developed to facilitate the calculation process. The GUI allows users to input coordinates, generate graphs, calculate sums, and ultimately determine the approximated area of the object. By leveraging the power of GUIs and visual aids, this research aims to enhance spatial understanding and support informed decision-making in various applications. The findings showcase the potential of the discrete least squares method as a valuable tool for accurate area estimation, paving the way for further advancements in this field.*

Keywords: discrete least square method, area aprroximation, Graphical User Inteface, AreaXpert.

DOI: 10.5281/zenodo.11044347

1. INTRODUCTION

The concept of area, a fundamental notion in mathematics, is used to measure the size of a two-dimensional surface or region (Tossavainen et al., 2016). It is typically expressed in square units such as square meters, square centimetres, or square feet. The calculation of area is crucial in various mathematical and real-world contexts, including geometry, calculus, physics, and engineering (Kumar, 2002). For irregular shapes, the area can be determined by dividing the shape into simpler ones such as triangles, rectangles, or circles. The areas of these simpler shapes are then calculated separately and summed to find the total area of the irregular shape (Lee, 2006).

The Discrete Least Square Method, often referred to as the least squares approximation, is a mathematical technique primarily used for estimating the actual value of a quantity while considering the errors present in observations or measurements (Ibrahim, 2022). This method holds significant relevance in data-fitting applications, especially when it comes to finding an appropriate mathematical function to represent real-world phenomena (Syarafina Mohamed et al., 2018). The goal of this process is to minimize the sum of squared residuals between the actual solution and the approximated solution at each point (Eason & Mote, 1977).

With the advent of Graphical User Interfaces (GUIs), human-computer interaction has transformed significantly by simplifying programming complexities and leveraging computers' graphical capabilities (Martinez, 2011). This project aims to explore the utilization of the Discrete Least Squares Method in approximating the area of Pulau Ketam, hence we propose the development of a simulator known as AreaXpert. The software will employ user-friendly GUIs and visual aids like graphs to provide a comprehensive tool for accurately estimating Pulau Ketam's area. By doing so, AreaXpert will contribute to improved spatial understanding and informed decision-making in various applications.

2. METHOD & MATERIAL

The material being referred to in the research is a detailed map of Pulau Ketam, an island located in Selangor, Malaysia. The map was sourced from Google Maps, as illustrated in Figure 1. Subsequently, the map was carefully plotted onto a graph paper, as depicted in Figure 2.

Figure 1. The Map of Pulau Ketam.

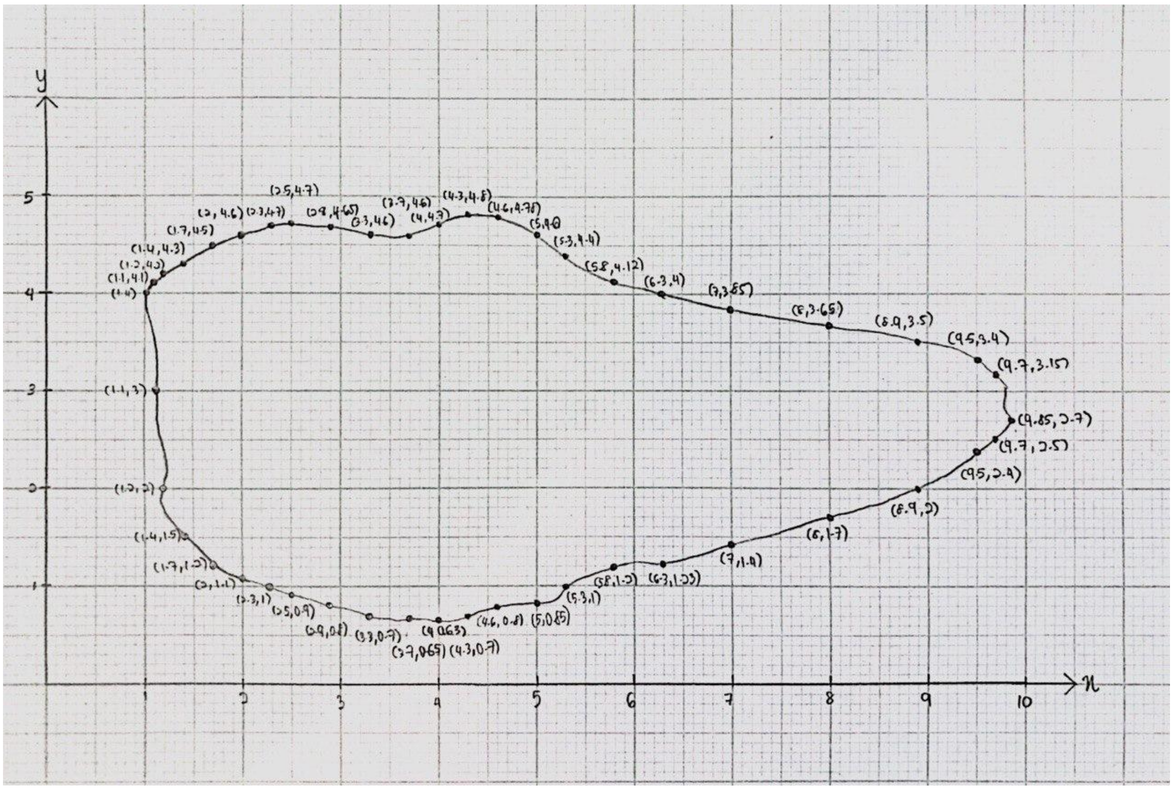

Figure 2. Pulau Ketam Map's plotted on the Graph Paper.

The App Designer was utilized to create the graphical user interface (GUI), offering an intuitive platform for developing behavior and interfaces. With an emphasis on convenience and accessibility, the GUI includes an extensive range of essential shapes for application design. Refer to Figures 3 to Figure 5 for visual representations of the interface.

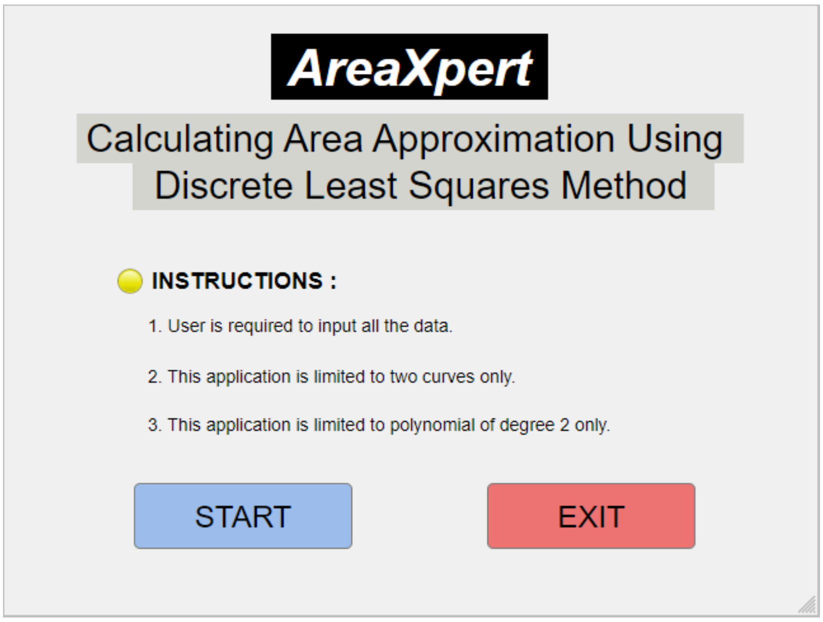

Figure 3. The first layout of the GUI

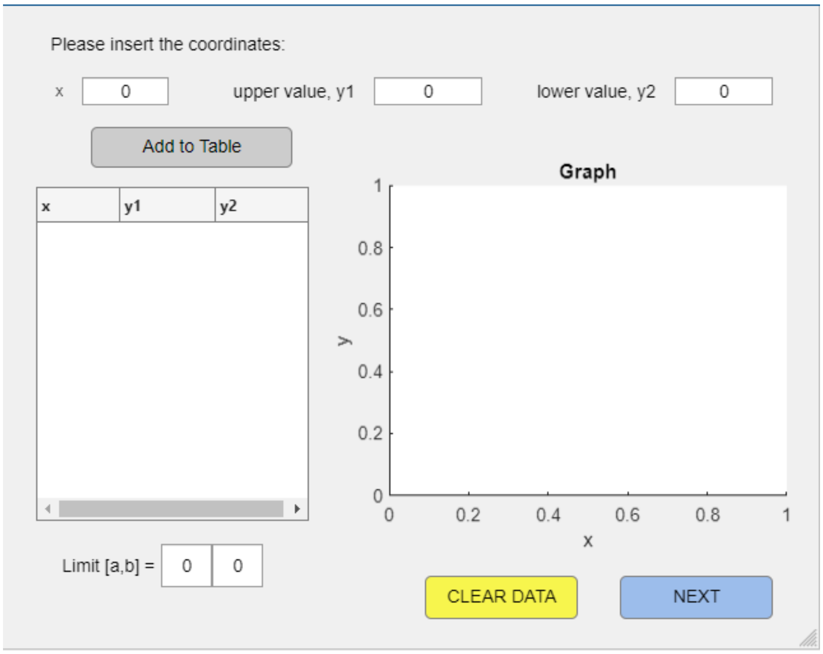

Figure 4. The second layout of the GUI

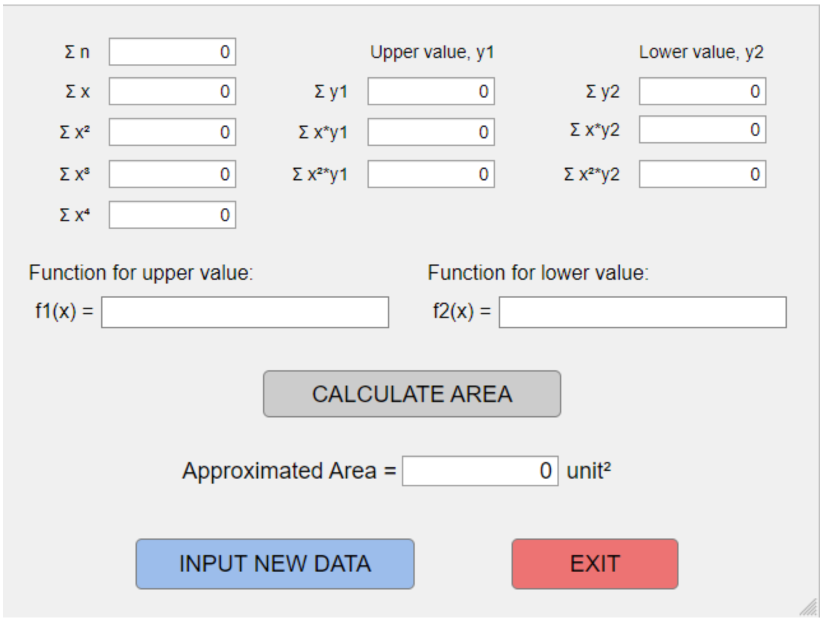

Figure 5. The third layout of the GUI

The GUI for approximating area using the Discrete Least Square method is designed to help users calculate the approximation of an area using given coordinates. A flowchart diagram is provided in Figure 4 to give users a clear picture of how the GUI will function. To start, users need to insert the values of $x1, y1$ ($upper\ value\ of\ y$), and $y2$ ($lower\ value\ of\ y$) to populate the data in the table. After that, users should insert the $limit$ $[a, b]$ to generate and display the graph of the given data. Users can then choose to continue by clicking on the "NEXT" button or re-enter the data by clicking on the "CLEAR DATA" button. If users choose to continue, the system will display all the sums needed for the Discrete Least Square formula. It will then generate the functions of $f1(x)\ and\ f2(x)$. Next, users need to click the "CALCULATE AREA" button for the system to calculate the area and produce the approximated area. If users want to insert new data, they can press the "INPUT NEW DATA" button, which will reset the data and allow them to enter new data. Alternatively, users can exit the system by clicking the "EXIT" button. This comprehensive GUI will help users to easily approximate area using the Discrete Least Square method. The details of the flowchart of the GUI are given in the Figure 6.

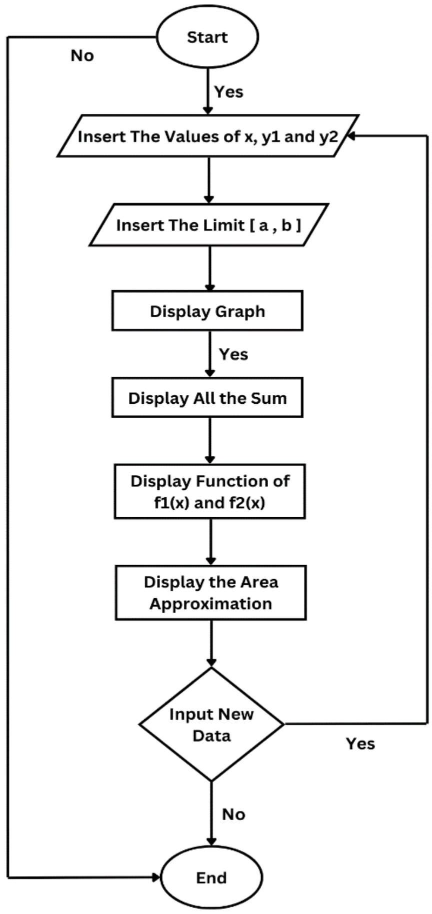

Figure 6. Flowchart of the GUI.

3. FINDINGS

The following report delves into the application of the discrete least square method in approximating the area of Pulau Ketam. The report presents 24 data points collected from Figure 2 and displayed in Table 1. In this context, the variable 'x' denotes the x-axis, while the variable 'y' represents the y-axis.

Table 1. The data points of Pulau Ketam.

x	*y1 (upper value of y)*	*y2 (lower value of y)*
1	4	4
1.1	4.1	3
1.2	4.2	2
1.4	4.3	1.5

1.7	4.5	1.2
2	4.6	1.1
2.3	4.7	1
2.5	4.7	0.9
2.9	4.65	0.8
3.3	4.6	0.7
3.7	4.6	0.65
4	4.7	0.63
4.3	4.8	0.7
4.6	4.78	0.8
5	4.6	0.85
5.3	4.4	1
5.8	4.12	1.2
6.3	4	1.23
7	3.85	1.4
8	3.65	1.7
8.9	3.5	2
9.5	3.4	2.4
9.7	3.15	2.5
9.85	2.7	2.7

3.1 Solution

Step 1: In the first interface, the user is required to read the instructions first and then click the 'START' button to continue. Figure 7 below shows step 1 of the interface.

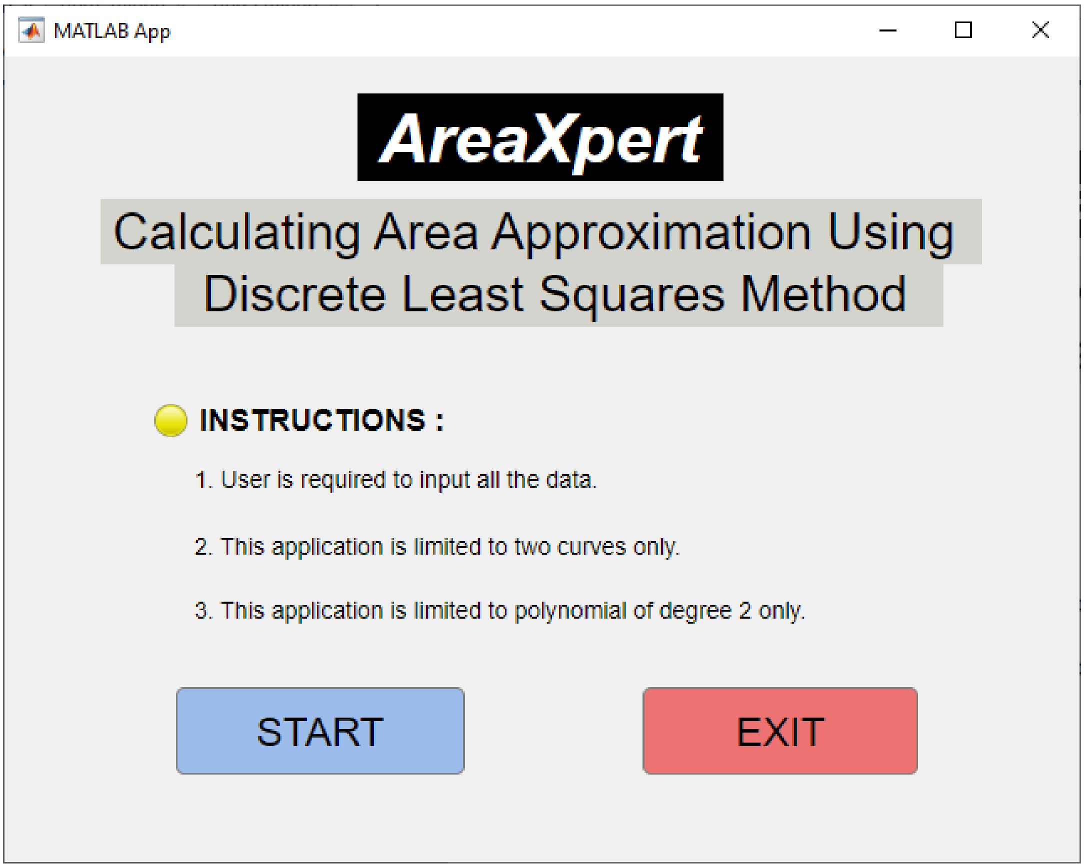

Figure 7. Step 1 - Initial Instructions.

Step2: In the second interface, user required to insert the coordinates of x, $y1$ ($upper\ value\ of\ y$) and $y2$ ($lower\ value\ of\ y$) then click "Add to table" button to save the coordinates in the table and also generate the graph. Then user need to insert the value of limit a and limit b. Click "NEXT" button to continue.

Figure 8 below shows the results obtained by inputting all the 24 data points for Pulau Ketam. The limit a is determined as the minimum value of x, which is 1. On the other hand, the limit b is the maximum value of x, which is 9.85. The graph consists of two lines, each with a distinct colour. The green line on the graph represents the $(x, y1)$ coordinates, while the blue line represents the $(x, y2)$ coordinates.

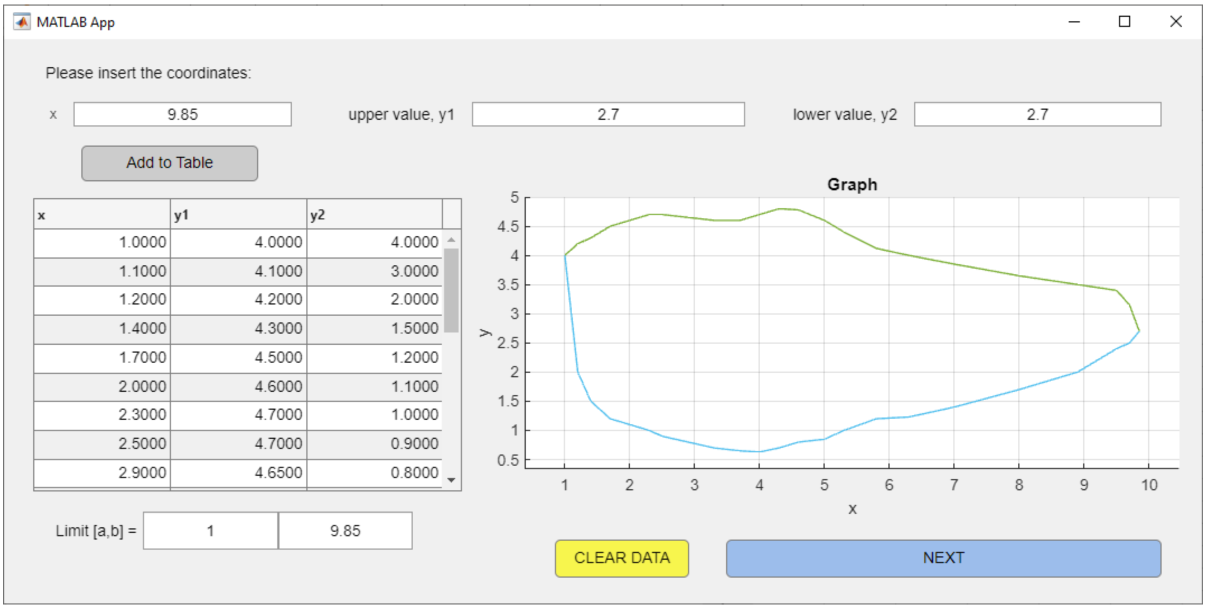

Figure 8. Step 2 - Coordinates Entry and Graph Visualization.

Step 3: In the third interface, the value of the sum of $n, x, x^2, x^3, x^4, y1, xy1, x^2y1, y2, xy2, x^2y2$ and the functions $f1(x)$ *and* $f2(x)$ will be displayed automatically. The displayed functions are polynomial degree 2, determined by using the formula of the discrete least squares method, where the general normal equations in matrix form (3×3) will be used as in equation (1).

$$\left[n \sum x \sum x^2 \sum x \sum x^2 \sum x^3 \sum x^2 \sum x^3 \sum x^4\right][a_0\ a_1\ a_2] = \sum y \sum xy \sum x^2y \tag{1}$$

The coefficients of $a_0, a_1, and\ a_2$ will be solved in equation (1) using Gauss-Jordan elimination method where it simplifies systems of linear equations using row operations. This process continues until the system is in reduced row echelon form, which makes it easy to obtain the solution (Sekhon & Bloom, 2020). The values of $a_0, a_1, and\ a_2$ will then be substituted into equation (2) to obtain least square functions.

$$f(x) = a_0 + a_1x + a_2x^2 \tag{2}$$

In Step 2, Pulau Ketam's graph has two lines of coordinates: one for $(x, y1)$ and another for $(x, y2)$. To calculate the coefficients of $a_0, a_1, and\ a_2$, two functions are derived for both sets of coordinates, resulting in two functions: $f1(x)$ for $(x, y1)$ coordinates and $f2(x)$ for $(x, y2)$ coordinates. A detailed representation of the values of all the sums is shown in Figure 9. The functions obtained for Pulau Ketam are $f1(x) = 3.9587 + 0.3413x - 0.0455x^2$ and $f2(x) = 3.2154 - 0.9902x + 0.0969x^2$.

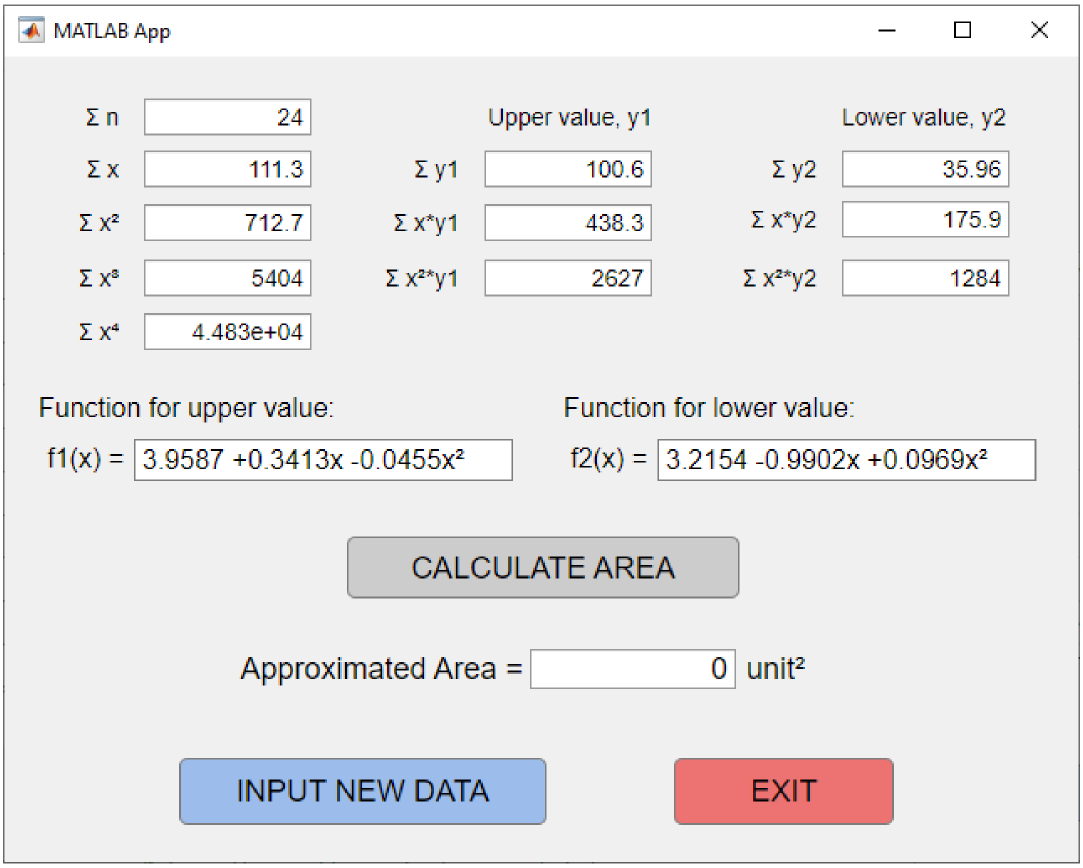

Figure 9. Step 3 - Deriving Least Square Functions.

Step 4: User required to click the "CALCULATE AREA" button to display the value of approximated area. The area will be calculated using area under the curve formula by integrating the least square function, $f1(x)$ *and* $f2(x)$ obtained in Step 3 and the value of limit a and b were obtained in Step 1.

The process of calculating the area under a curve and above the x-axis can be accomplished by using the definite integral. According to Green (2013), when we have two curves, $y = f1(x)$ *and* $y = f2(x)$, and $f1(x)$ *is greater than* $f2(x)$, the area between them which is bounded by the horizontal lines $x = a$ and $x = b$ can be determined using the formula area as shown in equation (3). Figure 10 shows the results of the approximated area of the Pulau Ketam which is 25.2 unit².

$$Area = \int_a^b f1(x) - f2(x)\, dx \tag{3}$$

$$Area = \int_1^{9.85} (3.9587 + 0.3413x - 0.0455x^2) - (3.2154 - 0.9902x + 0.0969x^2)\, dx$$

$$Area = 25.2\ unit^2$$

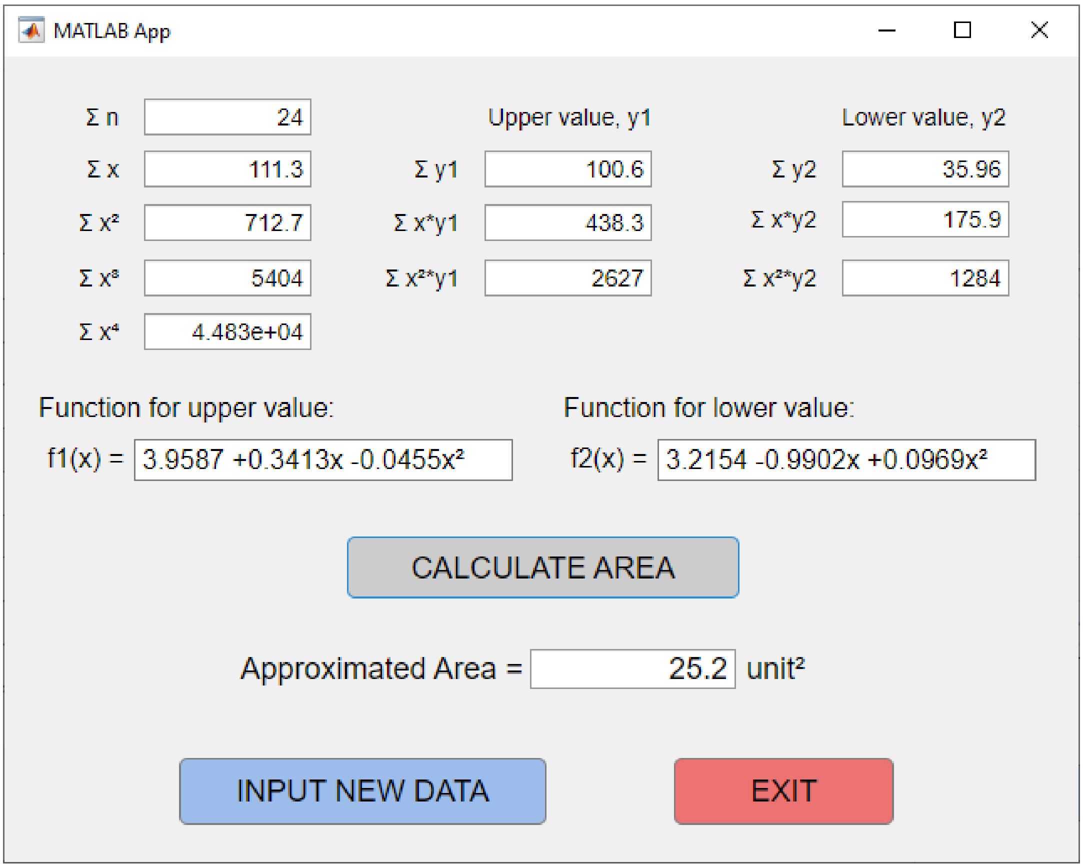

Figure 10. Step 4 - Computing Approximated Area.

3.2.2 Results

As a result, the manual calculation arrived at an area of 25.1901 unit², while AreaXpert produced a figure of 25.2 unit². The difference between the two outcomes is minimal, just 0.0099 unit². This disparity is due to rounding; AreaXpert rounds to one decimal place, while the manual calculation may have used additional decimal places in its intermediate calculations. Nevertheless, calculating the area manually is a complex and time-consuming task, especially when handling a large number of data points. In contrast, AreaXpert provides a more efficient and accurate approach to obtain the area in a timely and precise manner.

The map of Pulau Ketam on Google Maps displays a scale of 2 cm to 1 km (actual), which implies that for every 1 unit (on the graph), the distance covered is 2 cm. Therefore, if we consider 1 unit² (on the graph), it would be equal to 1 km² in actuality. Hence, we can conclude that the area of Pulau Kapas is estimated to be approximately 25.2 km².

According to the data provided by Google, the exact measurement of Pulau Ketam is 22.92 km². However, there seems to be a slight discrepancy between the estimated area by AreaXpert and the actual area of the Pulau Ketam, which could be due to some error. To determine the extent of this error, we can make use of the percentage relative error formula, $\varepsilon\%$ as follows (Tim, 2016):

$$\varepsilon\% = \left|\frac{Actual\ Value - Approximated\ Value}{Actual\ Value}\right| \times 100\%$$

$$\varepsilon\% = \left|\frac{22.92\ km^2 - 25.2\ km^2}{22.92\ km^2}\right| \times 100\% = 9.95\%$$

4. DISCUSSION

The discrete least square method has gained significant attention in recent years due to its numerous advantages, including its ability to accurately approximate functions, handle missing or noisy data, and perform efficient computations. To fully capitalize on these benefits, a user-friendly graphical interface called AreaXpert has been designed and developed. This interface enables faster and more optimal fitting of data points, thereby facilitating precise estimations of the area of an object. For instance, the area of irregularly shaped objects such as Pulau Ketam can be easily measured using AreaXpert. By simply entering all the data points into the interface, the system can produce an approximated area within a second. The AreaXpert is an incredibly useful tool for various industries and educational sectors, including educators, students, and researchers, who can benefit from its ability to accurately estimate the area of objects.

5. CONCLUSION

In conclusion, the utilization of the discrete least squares method in the AreaXpert simulator has proven to be a reliable method for determining the area of an object of irregular shape. This study employed the Discrete Least Squares Method and the area under the curve formula to approximate the area of Pulau Ketam, resulting in an estimated area of 25.2 units^2. The Discrete Least Squares Method, known for its versatility in fitting data points, proved instrumental in this calculation, showcasing its broader applicability beyond traditional data analysis tasks. This research contributes to the understanding and implementation of the Discrete Least Squares Method, providing valuable insights for researchers and practitioners across various domains seeking precise area estimations.

Acknowledgments: Gratitude is extended to the numerous individuals whose unwavering support and assistance significantly contributed to the successful completion of this project. Their encouragement and guidance were instrumental throughout the research journey. Special appreciation is also reserved for the organizations or institutions for providing the necessary resources and facilities that facilitated the research process.

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