EXPLORING NOVEL APPLICATIONS OF NEWTON FORWARD DIVIDED DIFFERENCE INTERPOLATION METHOD IN PERIMETER CALCULATION

Sidik Rathi[1, *], Nurzalina Harun[2], Azneen zuhreen Burhanud Din[3], Zubaidah Sadikin [4], Mohd Agos Salim Nasir [5]

[1]   Universiti Teknologi MARA Shah Alam; sidik8423@uitm.edu.my; ⓘD ORCID ID (https://orcid.org/0000-0002-6408-3920)

[2]   Universiti Teknologi Mara Shah Alam; nurzalina1587@uitm.edu.my; ⓘD ORCID ID (https://orcid.org/0000-0003-0477-580X)

[3]   Universiti Teknologi Mara Shah Alam; 2021494292@student.uitm.edu.my; ⓘD ORCID ID (https://orcid.org/0000-0003-4071-9581)

[4]   Universiti Teknologi MARA Shah Alam; zubaidah1590@uitm.edu.my; ⓘD ORCID ID (https://orcid.org/0000-0003-3967-5243)

[5]   Universiti Teknologi Mara Shah Alam; mohdagos066@uitm.edu.my; ⓘD ORCID ID (https://orcid.org/0000-0002-8761-4595)

*   Correspondence: mohdagos066@uitm.edu.my; 60183845172

Abstract:

This study addresses the challenge of calculating the perimeter of irregular shapes, particularly for individuals without a strong mathematical background. The study involves developing a user-friendly simulator, PeriTech that simplifies the process of perimeter calculation by providing a visually intuitive interface. This study focuses on calculating the perimeter of irregular shape objects such as piper betle leaf. The method involves developing a mathematical equation that corresponds to the boundary of the piper betle leaf to calculate the perimeter of the leaf. Newton Forward Divided Difference Interpolation is used to find a polynomial equation from the data points. With this approach, an object's form may be divided into basic and more simple curve shapes, each of which's arc length can be calculated, and combine the arc lengths to determine the estimated perimeter.

Keywords: Newton Forward Divided Difference Interpolation, Arc Length, Simulator, Piper Betle

1. INTRODUCTION

Perimeter is the outer edge of an area of land or the border around it (Russ et al., n.d.). In other words, it refers to the total length or distance around the outer boundary of an object. The shape of the boundary can be formed in various shapes such as rectangles, circles, and triangles. In the context of the real world, the shape of an object is usually in the categorical as complex and irregular shape. Finding the perimeter for irregular shapes is by breaking the shape down into simpler shapes or in other words breaking it into several sectors and adding up each of them.

Arc length is the distance along the curve of an arc. It is the length of the curve between two points on the curve. In calculus, the arc length formula is used to find the length of a curve between

two points. The measurement of the perimeter of an irregular object can be calculated by combining the value of the arc length for each curve(Walter & Fournier, 2014).

A mathematical approach called Newton's Divided Difference Interpolation is used to approximate the value of a function at a position between two known values. It is a numerical algorithm used for polynomial interpolation of a set of data (Srivastava & Srivastava, n.d.). This numerical method involves constructing a polynomial that passes through given points for approximating a function. The basic idea behind the method is to recursively compute a set of coefficients called divided differences that determine the coefficients of the interpolating polynomial. There are two types of Newton's Divided Difference Interpolation method which are Newton Forward Divided Difference Interpolation and Newton Backward Divided Difference Interpolation.

This study is focused on calculating the perimeter of the piper betle leaf. A leaf is one of a real-world object that has an irregular shape. One of the important characteristics to identify a plant is its leaf. This study aims to find the approximation of the perimeter and the difference between the exact value and the estimated value. The perimeter of a leaf is the continuous line forming the boundary of the leaf shape. It is measured as the total length of the leaf's outer edge (Rajeev, n.d.). The reason why this study is focused on finding the perimeter of piper betle lead is because piper betle is a valuable plant that is well-known for its medical properties and has been traditionally used in various cultural practices and traditional medicine systems. The leaves of Piper betle are heart-shaped with a pointed apex and emit a sharp, pungent aroma (Azahar et al., 2020.). Knowing the perimeter of the piper betle can help in recognizing the best leaf size that is suitable for medicine.

A simulator is a specialized type of simulation tool that involves the use of specific software to simulate a task, practice, or experimentation (Chernikova et al., 2020). Simulators can help predict outcomes, test hypotheses, optimize designs, train individuals, and analyze scenarios in controlled and virtual environments by running simulations based on mathematical models and input parameters. Simulators typically feature interactive elements that allow users to manipulate variables, observe the outcomes, and receive feedback based on their actions. There are various fields that simulators can used including engineering, science, training, education, and entertainment. In education, the simulator provides several benefits for both students and teachers, especially in the context of mathematics education. Based on the article from Bush et al., (2018), the simulator helps students see the practical applications of mathematical concepts and understand their relevance outside the classroom. Besides, the function terminology and the relationship between variables, such as X and Y, are easier to comprehend with this visual approach than in the simulator.

In this project of study, the PeriTech simulator will calculate the perimeter estimation by using the Newton Forward Divided Difference Interpolation method. The presence of a simulator can make the perimeter calculation for irregular shape of object procedure easier. The estimated perimeter of the piper betle leaf will be calculated by PeriTech. The actual perimeter of piper betle and the estimated perimeter will be compared to define the accuracy of the simulator and the method of Newton Forward Divided Difference Interpolation. MATLAB R2023b software was used in the development of this simulator.

2. METHOD & MATERIAL

This study involves the following steps:

Step 1: Data Collection

This study computes the arc length of a piper betle leaf to determine the leaf perimeter, as shown in Figure 1. The image was plotted on graph paper, as shown in Figure 2, and plotted in Microsoft Excel, as shown in Figure 3. The leaf shape is divided into two parts, the upper and the lower

part of the leaf. Each part of the leaf is divided into several basic and more simple curve shapes. Figures 4 and 5 show the image of each part of the leaf and the divided curves. The collection of Data A is the data points of each curve from the upper part of the leaf and Data B is the collection of data points from the lower part of the leaf.
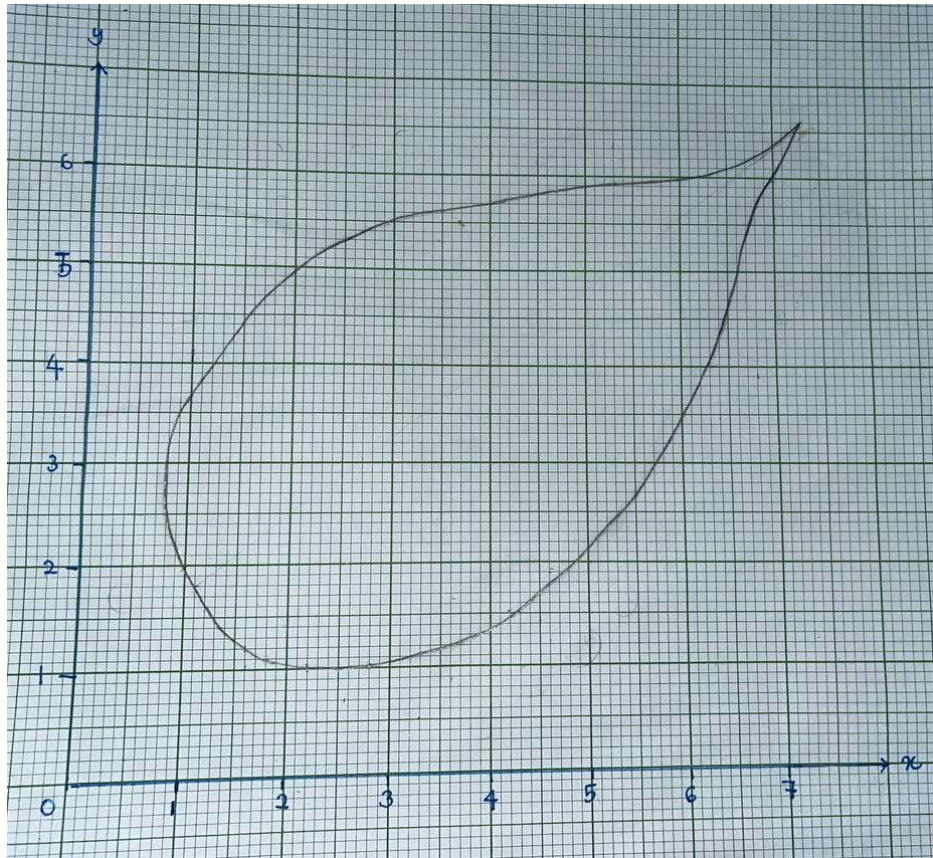

Figure 1: Piper Betle leaf image.
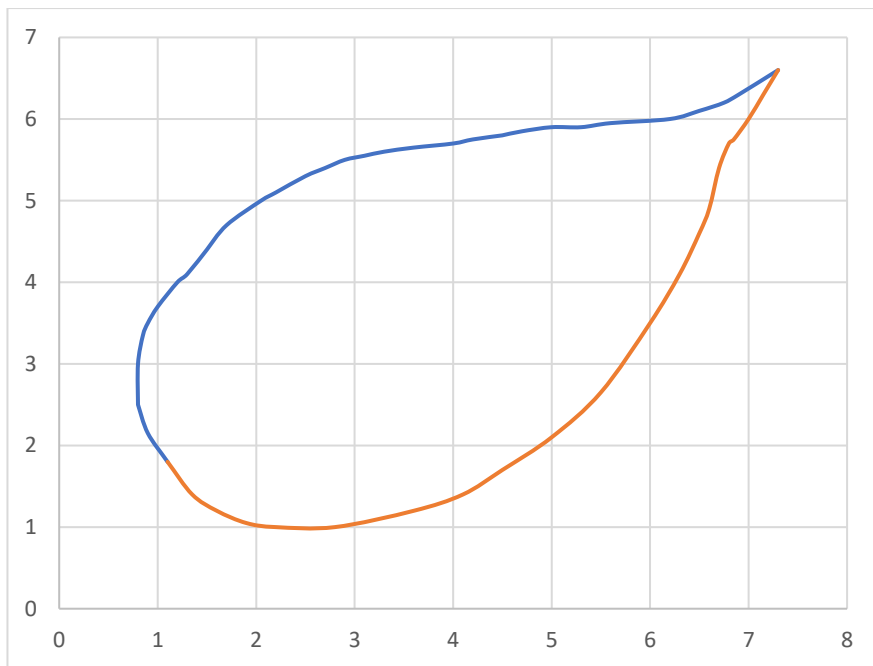
Figure 2: Leaf plotted in graph paper.

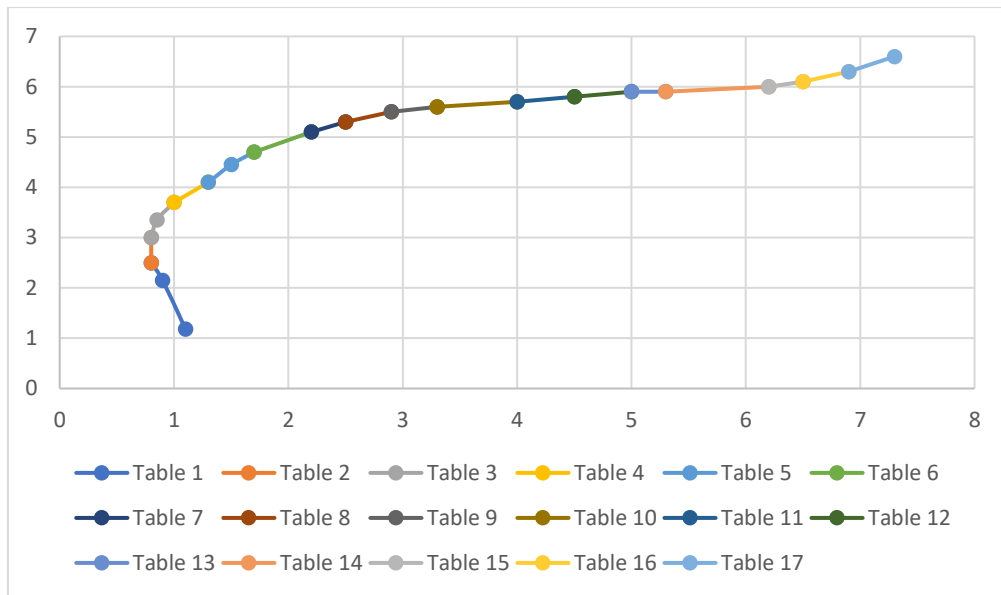Figure 3: Leaf plotted in Microsoft Excel.

Figure 4: The upper part of the leaf.

Data A: Set of data points from the upper part of the leaf.

Table 1	
x	y
0.8	2.5
0.9	2.15
1.1	1.18

Table 2	
x	y
0.8	2.5
0.8	3

Table 3	
x	y
0.8	3
0.85	3.35
1	3.7

Table 4	
x	y
1	3.7
1.3	4.1

Table 5	
x	y
1.3	4.1
1.5	4.45
1.7	4.7

Table 6	
x	y
1.7	4.7
2.2	5.1

Table 7	
x	y
2.2	5.1
2.5	5.3

Table 8	
x	y
2.5	5.3
2.9	5.5

Table 9	
x	y
2.9	5.5
3.3	5.6

Table 10	
x	y
3.3	5.6
4	5.7

Table 11	
x	y
4	5.7
4.5	5.8

Table 12	
x	y
4.5	5.8
5	5.9

Table 13	
x	y
5	5.9
5.3	5.9

Table 14	
x	y
5.3	5.9
6.2	6

Table 15	
x	y
6.2	6
6.5	6.1

Table 16	
x	y
6.5	6.1
6.9	6.3

Table 17	

x	y
6.9	6.3
7.3	6.6
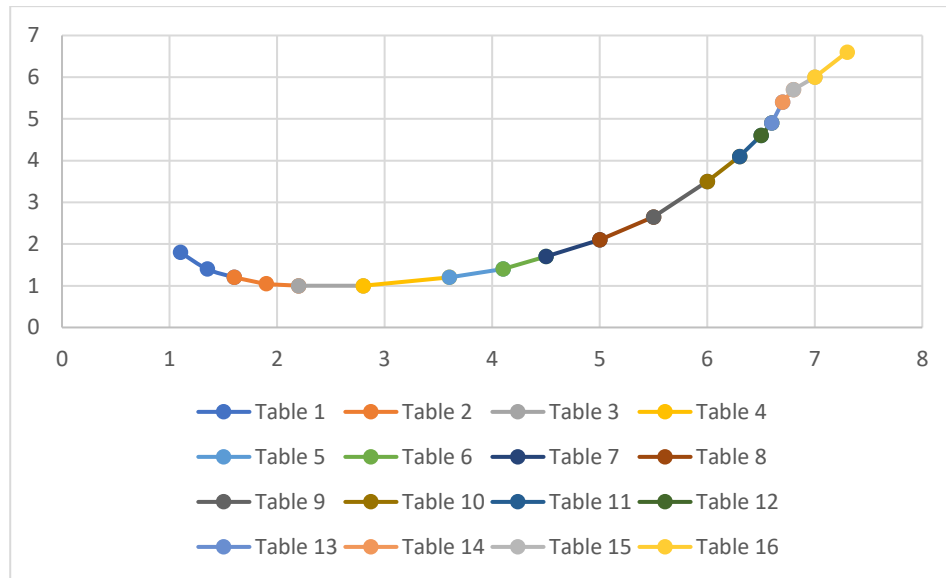
Figure 5: The lower part of the leaf.

Data B: Set of data points from the lower part of the leaf.

Table 1	
x	y
1.1	1.8
1.35	1.4
1.6	1.2

Table 2	
x	y
1.6	1.2
1.9	1.05
2.2	1

Table 3	
x	y
2.2	1
2.8	1

Table 4	
x	y
2.8	1
3.6	1.2

Table 5	
x	y
3.6	1.2
4.1	1.4

Table 6	
x	y
4.1	1.4
4.5	1.7

Table 7	
x	y
4.5	1.7
5	2.1

Table 8	
x	y
5	2.1
5.5	2.65

Table 9	
x	y
5.5	2.65
6	3.5

Table 10	
x	y
6	3.5
6.3	4.1

Table 11	
x	y
6.3	4.1
6.5	4.6

Table 12	
x	y
6.5	4.6
6.6	4.9

Table 13	
x	y
6.6	4.9
6.7	5.4

Table 14	
x	y
6.7	5.4
6.8	5.7

Table 15	
x	y
6.8	5.7
7	6

Table 16	
x	y
7	6
7.3	6.6

Step 2: Perimeter Estimation.

This step involves computing the polynomial equation, calculating the arc length, and adding the arc length to find the approximate perimeter. This study used the Newton Forward Divided Difference Interpolation method to find the polynomial equation of each curve of the parts of the leaf. The value of x represents the distinct values and y represents the corresponding function values. All the data points need to be arranged in ascending value to follow the order of x-values. Figure 6 is part of the interface of PeriTech where the user needs to insert the coordinates of x and y into the table. This PeriTech only accepts two or three sets of data points because this simulator can compute polynomial degree one and degree two. The formula Newton Forward Divided Difference Interpolation for degree one is equation (1) and degree two is equation (2).

$$P_1 = f[x_0] + f[x_0, x_1](x - x_0) \tag{1}$$

$$P_2 = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) \tag{2}$$

Where finite divided difference can be calculated using equation (3), (4), and (5).

$$f[x_i] = y_i \tag{3}$$

$$f[x_i, x_{i+1}] = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} - x_i} \tag{4}$$

$$f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_i, x_{i+1}]}{x_{i+2} - x_i} \tag{5}$$
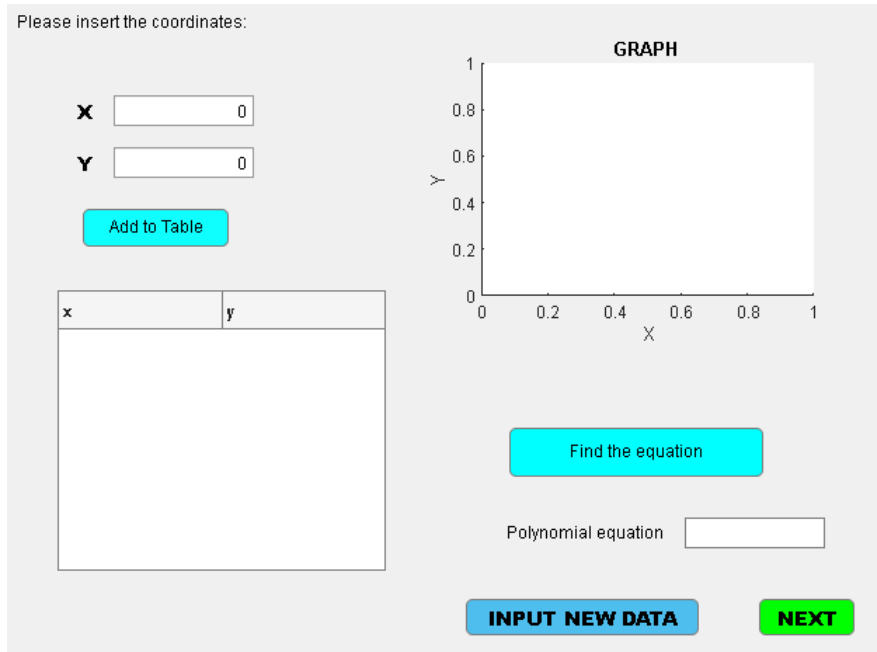
Figure 6

The arc length of each curve will be calculated using equation (6). The Figure 7 is the interface of the PeriTech to compute the arc length where the users need to insert the polynomial equation, limit a, and limit b.

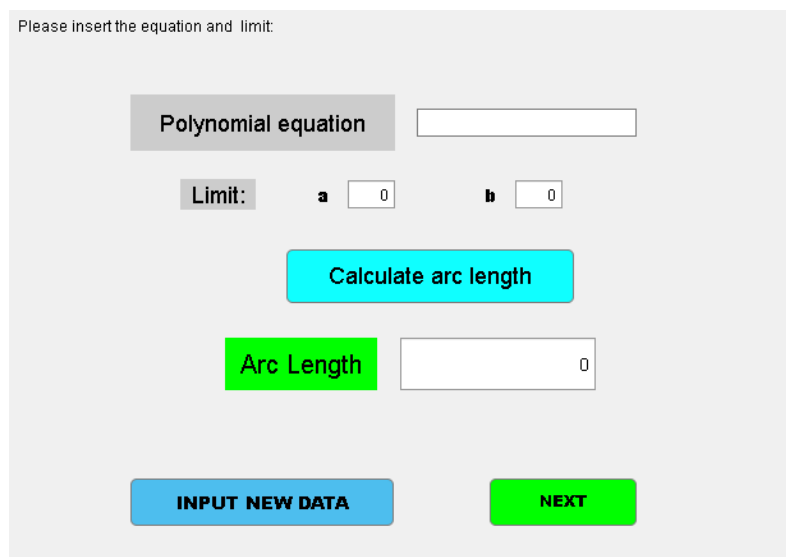$$L = \int_a^b \sqrt{1 + (P'(x))^2} dx \qquad (6)$$

Figure 7

The estimate perimeter can be obtained by totalling up all the value of the arc lengths of each curve. Figure 8 show the PeriTech interface to calculate the value of perimeter approximation. The

users only need to insert all value of the arc length for each polynomial equation into the PeriTech.

Figure 8

Step 3: Validation and accuracy

The difference between the value of the approximate perimeter by PeriTech simulator and manually calculated is used to validate the performance of the simulator. To evaluate the accuracy and reliability of the method used the percentage relative error is calculated by using equation (7). The exact perimeter is obtained using a thread and ruler while, the approximate perimeter is obtained from the PeriTech.

$$\varepsilon \% = \left| \frac{exact\ perimeter - approximate\ perimeter}{exact\ perimeter} \right| \times 100 \% \qquad (7)$$
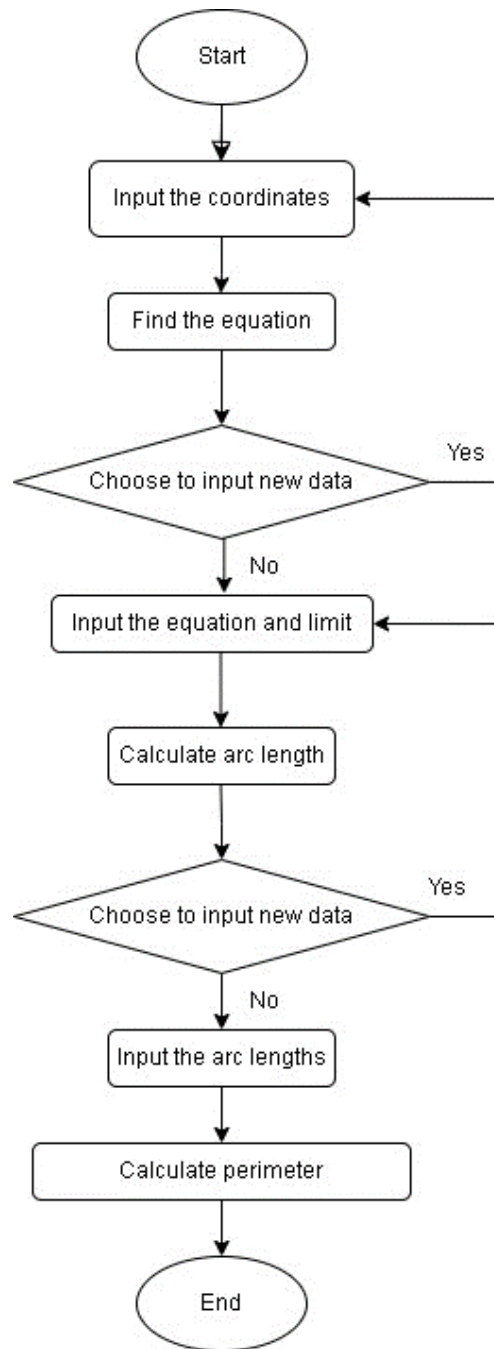
Figure 9: Flowchart of PeriTech

Figure 9 shows the flowchart diagram that presents the flow of a system. This flowchart diagram is the flowchart of the PeriTech simulator for approximate perimeter using Forward Newton Divided Difference Interpolation method.

3. FINDINGS

This study discusses the use of Newton Forward Divided Difference Interpolation method to find the perimeter of an irregular shape object by developing the PeriTech simulator via MatLab R2023b. In this study, the irregular shape object that was used is a piper betle leaf. Finding the perimeter of piper betle leaf is the focus of this study.

3.1 Solution

The leaf's surface was plotted and divided into two parts. The calculation can start from any part of the leaf and any set of data points from Data A or Data B.
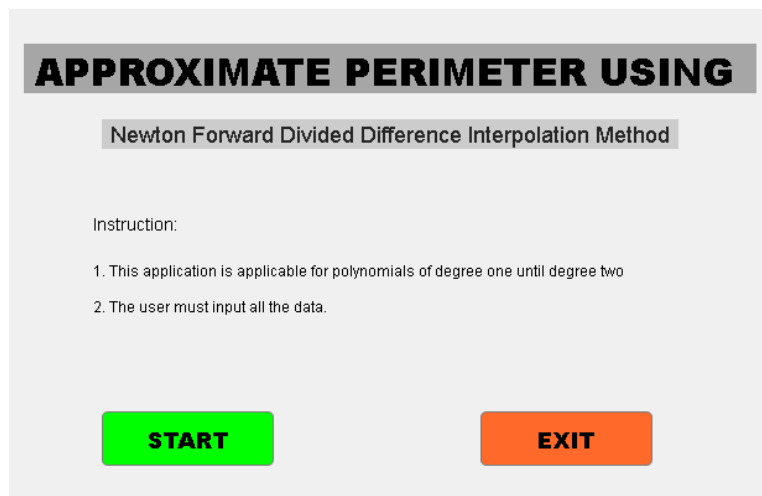

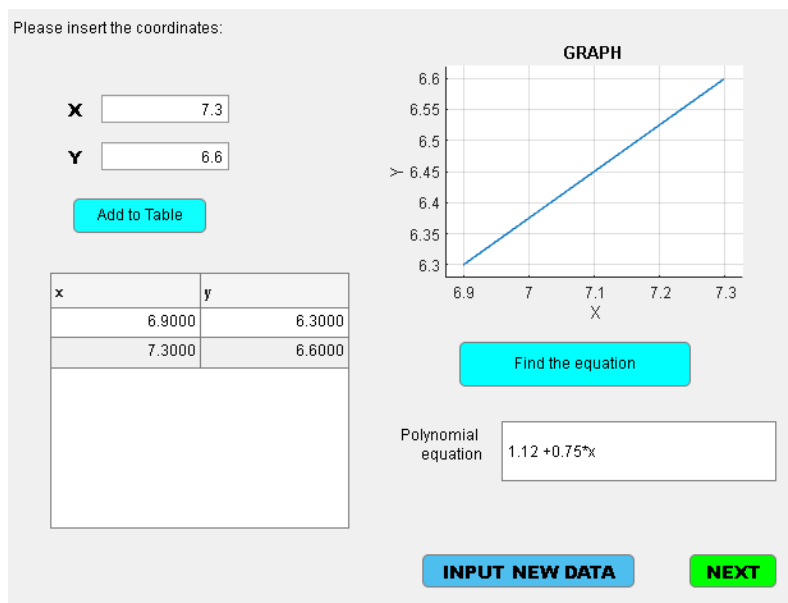
Figure 10: First interface of PeriTech

Figure 11: The polynomial equation for set of data points inserted is from Table 17 in the upper part of the leaf.
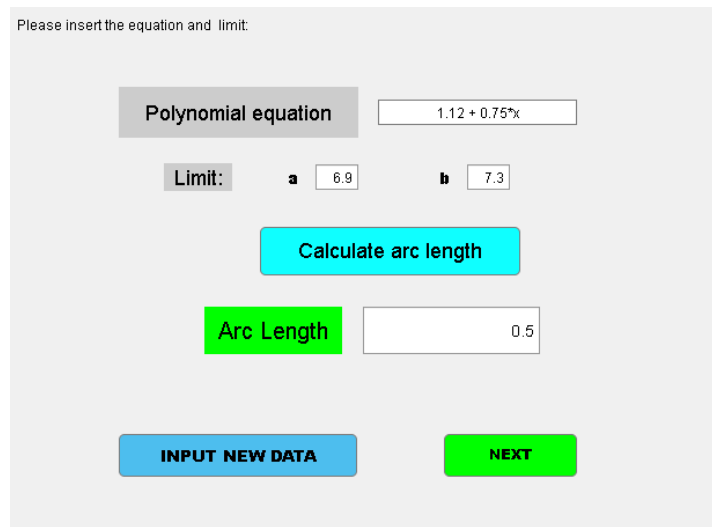
Figure 12: The value of arc length for set data in Table 17.

3.2 Result

Table bellow show the result of the arc length for each curve in both part of the leaf. There are two type of result which is from the manual calculation and from the PeriTech.

Table i: The value arc length for the upper part of the leaf

Data A: The Upper Part of the Leaf					
Table	Polynomial Equation	Limit		Arc Length	
		a	b	PeriTech	Manually
1	$y = -4.5x^2 + 4.15x + 2.06$	0.8	1.1	1.355	1.3547
2	$x = 0.8$	2.5	3	0.5	0.5
3	$y = 6.7x^2 - 9.1x + 6.73$	0.8	1.0	0.627	0.627
4	$y = 1.33x + 2.37$	1.0	1.3	0.4992	0.4992
5	$y = -1.25x^2 + 5.25x - 0.61$	1.3	1.7	0.724	0.7240
6	$y = 0.8x + 3.34$	1.7	2.2	0.6403	0.6403
7	$y = 0.67x + 3.63$	2.2	2.5	0.3611	0.3611
8	$y = 0.5x + 4.05$	2.5	2.9	0.4472	0.4472
9	$y = 0.25x + 4.78$	2.9	3.3	0.4123	0.4123
10	$y = 0.14x + 5.13$	3.3	4.0	0.7068	0.7068
11	$y = 0.2x + 4.9$	4.0	4.5	0.5099	0.5099
12	$y = 0.2x + 4.9$	4.5	5.0	0.5099	0.5099
13	$y = 5.9$	5.0	5.3	0.3	0.3
14	$y = 0.11x + 5.31$	5.3	6.2	0.9054	0.9054
15	$y = 0.33x + 3.93$	6.2	6.5	0.3159	0.3159
16	$y = 0.5x + 2.85$	6.5	6.9	0.4472	0.4472
17	$y = 0.75x + 1.12$	6.9	7.3	0.5	0.5

Table ii: The value arc length for the lower part of the leaf

Data B: The Lower Part of the Leaf					
Table	Polynomial Equation	Limit		Arc Length (cm)	
		a	b	PeriTech	Manually
1	$y = -0.96x^2 + 2.03x + 0.76$	1.1	1.6	0.5863	0.5863
2	$y = 0.56x^2 + 2.44x + 3.69$	1.6	2.2	2.806	2.8058
3	$y = 1.0$	2.2	2.8	0.6	0.6
4	$y = 0.25x + 0.3$	2.8	3.6	0.8246	0.8246
5	$y = 0.4x - 0.24$	3.6	4.1	0.5385	0.5385
6	$y = 0.75x - 1.67$	4.1	4.5	0.5	0.5
7	$y = 0.8x - 1.9$	4.5	5.0	0.6403	0.6403
8	$y = 1.1x - 3.4$	5.0	5.5	0.7433	0.7433
9	$y = 1.7x - 6.7$	5.5	6.0	0.9862	0.9862
10	$y = 2x - 8.5$	6.0	6.3	0.6708	0.6708
11	$y = 2.5x - 11.65$	6.3	6.5	0.5385	0.5385
12	$y = 3x - 14.9$	6.5	6.6	0.3162	0.3162
13	$y = 5x - 28.1$	6.6	6.7	0.5099	0.5099
14	$y = 3x - 14.7$	6.7	6.8	0.3162	0.3162
15	$y = 1.5x - 4.5$	6.8	7.0	0.3606	0.3606
16	$y = 2x - 8$	7.0	7.3	0.6708	0.6708

The Figure 13 show the result of perimeter approximation by PeriTech for perimeter of piper betle leaf. The Table the value of perimeter approximation for both methods. The ratio of the value of 1 unit on graph paper and the value of 1 unit cm is 1:2.

Figure 13: The value of perimeter approximation form PeriTech

Table iii: The value of perimeter approximation

Method	perimeter approximation	
	in graph paper	in cm
PeriTech	21.37	42.74
Manually	21.3688	42.7376

The result perimeter approximation from PeriTech and manually showing a small difference where PeriTech give the result into 2 decimal places. If the manually calculation is rounded the result for both methods show equally same to each other. Let assume the perimeter approximation is 42.74cm. The exact perimeter of leaf when measured by thread is 37.8cm. The percentage relative error obtained for this study as below:

$$\varepsilon\% = \left| \frac{exact\ perimeter\ -\ approximated\ perimeter}{exact\ perimeter} \right| \times 100$$

$$\varepsilon\% = \left| \frac{37.8\ cm\ -\ 42.74\ \ cm}{37.8\ cm} \right| \times 100\% = 11.56\%$$

4. DISCUSSION

PeriTech and the manual calculation result show that the PeriTech is the best way to determine the perimeter approximation. This is because it will become more difficult to do manual calculations since many curves that need to be calculated. It is time-consuming to find the arc length for each curve one by one. PriTech will reduce the calculation works and it will be easy and quick to determine the value of arc length and perimeter approximation. The result obtained from percentage relative error shows a good result where the error is 11.56%. Since the error is small less than 50%, indicates that the method of Newton Forward Divided Difference Interpolation method is a suitable method to find the perimeter approximation.

5. CONCLUSION

In conclusion, developing a simulator to calculate the perimeter gives many benefits to everyone from different fields, and not only to mathematicians who can get the benefits from its development. Other than can helping in reducing time-consuming, the simulator also gives visual representation and is user-friendly which can be easy to use for people without a strong mathematical foundation. Based on the result, given the small percentage relative error, the approximate perimeter calculated using the arc length equation is remarkably close to the exact value. Consequently, employing the Newton Forward Divided Difference Interpolation method proves to be an effective approach for determining the perimeter of an irregular shape object. This interpolation method enables the researchers to approximate the curve of the object's boundary accurately, allowing for precise measurements of its perimeter. This method provides a reliable estimation of irregularities in perimeter objects by dividing the object into smaller intervals and constructing a polynomial equation that passes through the data points. Therefore, utilizing a simulator and Newton Forward Divided Difference Interpolation method is a beneficial strategy in the approximately measured perimeter for irregular shape objects.

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