Research Article

UTILIZE THE LEAST SQUARE METHOD TO QUANTIFY THE PERIMETER OF GASTROINTESTINAL STROMAL TUMOUR

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Abstract:

This study focuses on calculating the arc length of gastrointestinal stromal tumours (GISTs) to determine the tumour's perimeter using the least-squares method. The method involves developing a mathematical equation that corresponds to the tumour model to calculate the perimeter even under uneven conditions. Least squares is used to find the best-fit line or curve that minimizes the sum of squared differences between observed data points and predicted values, commonly used in regression analysis. For tumour perimeter calculation from ultrasound data, the study employs both the least squares method and the Gauss-Jordan elimination method. By determining that 1 mm on the graph paper corresponds to 1.2929 mm in actual size, the total arc length is calculated to be 9.5026 mm. Manual measurements, especially with complex shapes and small details, can be prone to errors. Therefore, PeriSolve, a graphical user interface (GUI) and professional simulator, is designed to simplify perimeter calculations for uneven surfaces, with a specific focus on using the least squares method.

Keywords: Least square method, Gauss-Jordan elimination method, PeriSolve

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1. Introduction

Arc length is the perimeter between two points along a section of a curve (Calculating Arc Length, n.d.). At each step, the arc-length method takes a detour from the expected path, ensuring it does not get stuck by going straight (Fafard & Massicotte, 1993). Arc length offers a more accurate measurement of the perimeter between two points along a curved path. When analysing a tumour's surface or any curved structure, measuring perimeters in a straight line can result in notable inaccuracies. Arc length on a tumour surface represents the length of a curved line connecting two points.

This research calculates the arc length of a gastrointestinal stromal tumour (GIST) to determine the tumour's perimeter. The arc length can be computed using the least-squares method. The least-squares method is a mathematical regression analysis technique used to find the line that best fits a given dataset. The least-squares method visually illustrates the relationship between data points (Kenton, W., 2021) and is extensively applied across diverse fields such as statistics, economics, engineering, and data analysis. In physics and experimental sciences, the least-squares method (LSM) is a common choice for fitting purposes. Its statistical properties, which yield unbiased results with minimal variation regardless of the data's probability distribution function, contribute to its broad adoption. Furthermore, the solutions obtained through the least-squares method (LSM) tend to asymptotically converge towards a multivariate normal distribution (Helene et al., 2016). The least squares method is the method commonly used for generating a polynomial equation from a given data set (f(x) = a0 + a1x + a2x +...+ axe, where m < n). The objective of this study is to answer for the coefficients a0, a1, a2,... using the least-squares strategy by solving this system of equations using some of the techniques, such as the Gauss-Jordan elimination method. Gaussian elimination and Gauss-Jordan methods are two popular techniques utilized for solving systems of linear equations (Roshana Ali Naqvi et al., n.d.).

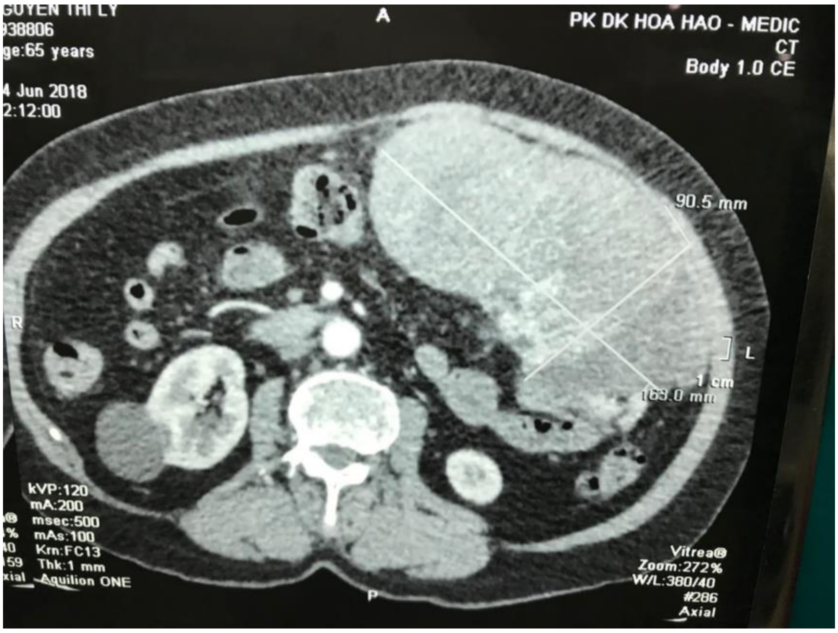
A simulator is a software system that emulates real-life scenarios or processes. It enhances user interaction by offering a user-friendly interface, often visual, where users can manipulate symbolic representations instead of typing commands line by line (Depcik & Assanis, 2005). Additionally, simulators provide a consistent method for navigating various applications through visual cues, making them self-explanatory and easy to use (Sherrick, n.d.). In this project, a simulator for approximating roots was developed to demonstrate how a0, a1, and a2 are determined based on a quadratic function and a given interval using the least squares method. Graphs were utilized as visual aids to provide users with a deeper understanding. The development of this simulator involved the use of mathematical software, specifically MAPLE 2016, to enhance project efficiency and progress. The project aimed to create PeriSolve, a simulator for approximating arc length using the least squares method, with additional features such as function graphs to enhance interactivity.

2. METHOD & MATERIAL

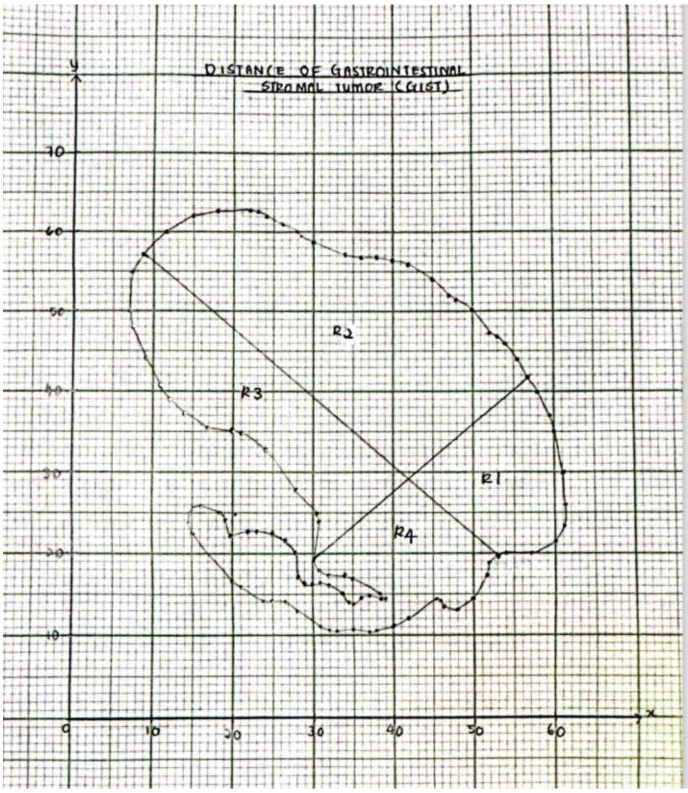
This study computes the arc length of a gastrointestinal stromal tumour (GIST) to determine the patient's tumour's perimeter from an ultrasound image, as shown in Figure 1A. The image was plotted on graph paper, as shown in Figure 1B.

Figure 1: Whole Gastrointestinal Stromal Tumour (GIST)
(This data on ultrasound is taken from an online website, https://www.ultrasoundmedicvn.com/2018/06/case-500-big-gist-tumour-dr-phan-thanh.html)

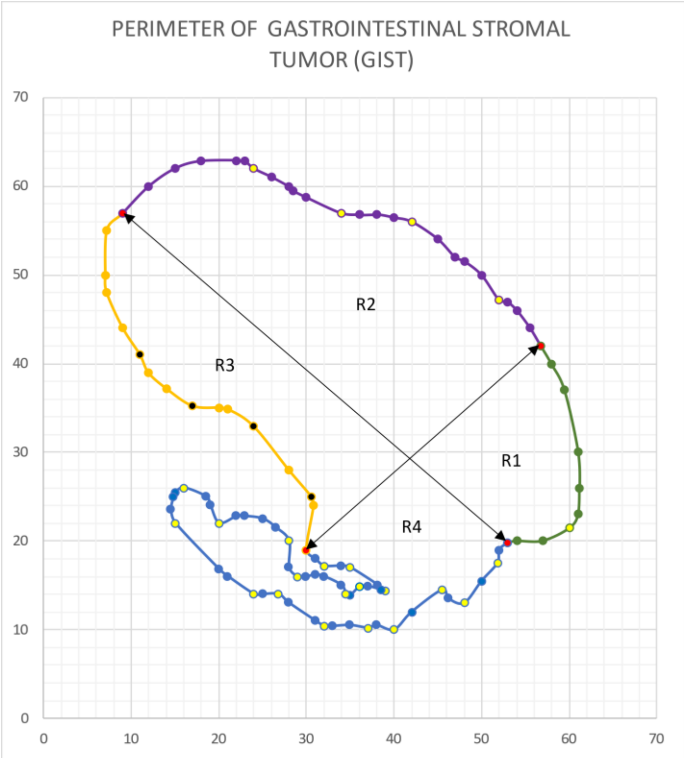
A: Original Ultrasound Image



B: Tumour plotted in graph paper



C: Tumour plotted in Microsoft Excel



The surface of the tumour is divided into four regions, and each region is divided into several parts.

The study utilized the least-squares method to compute the arc length of a tumour. This method, commonly employed in regression analysis, identifies the best-fit line or curve by minimizing the sum of squared differences between observed data points and predicted values. It was chosen for its effectiveness in establishing relationships between variables and making predictions. While linear regression is effective for fitting data to a line, higher-degree polynomials may be more appropriate in certain cases. The process of regression analysis begins with plotting data points on a graph and selecting the most suitable model. This iterative process can be challenging, leading to calculation errors and requiring careful verification. However, the development of a simulator for arc length problems can aid students by providing a tool to check their solutions quickly and accurately. By inputting relevant information, students can obtain the solution without the need for complex manual calculations. This simulator offers a unique feature designed specifically for determining arc lengths.

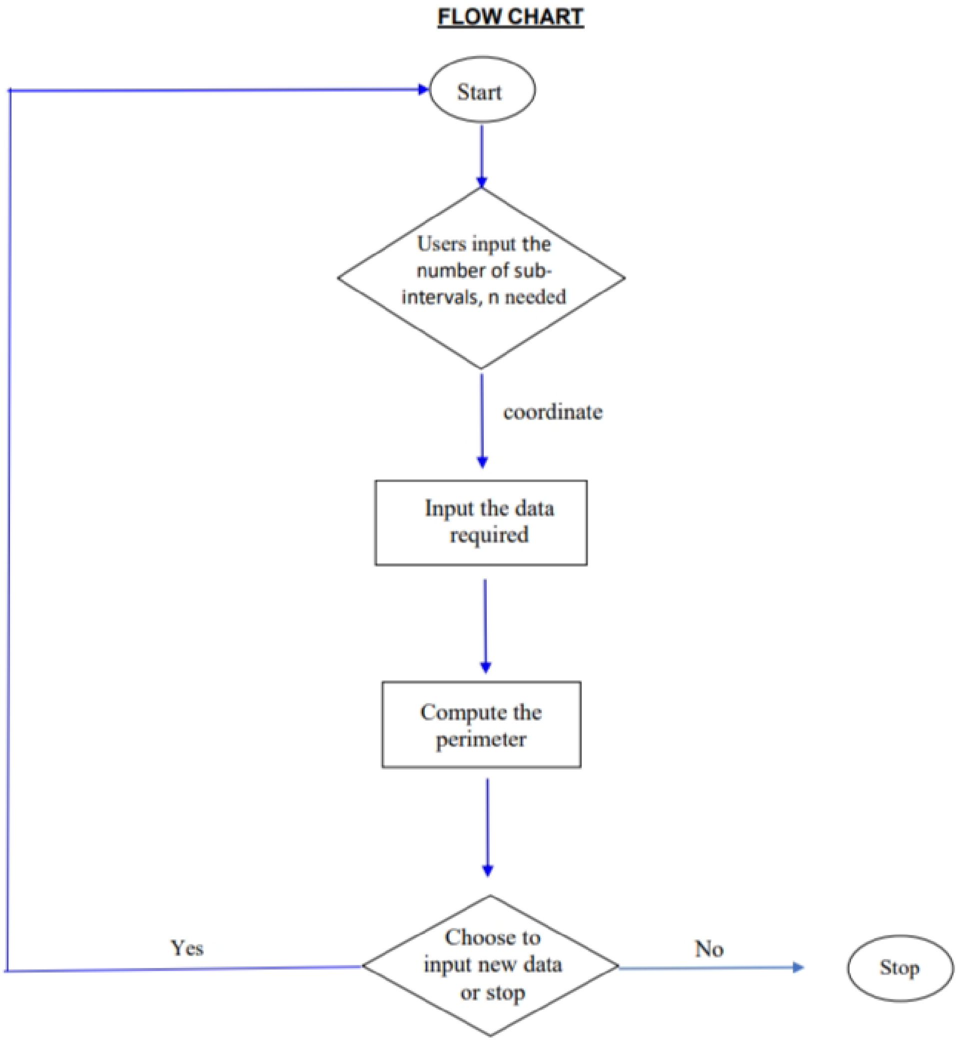


Figure 2. Flowchart of the Procedure.

3. FINDINGS

The findings will be explained in this section. In this study, the perimeter of a tumour is the main focus. The arc length of a gastrointestinal stromal tumour (GIST) is chosen as a sample to determine the patient’s tumour's perimeter for the purpose of this study.

The tumour’s surface was plotted on graph paper, and the calculations obtained are between coordinates (53, 19.8) and (60, 21.5), as shown in Figure 3. The full surface of the tumour was divided into four regions, and each region will be divided into several parts, as shown in Figure 1C. The calculation starts in region 1. In Region 1, the region was divided into two parts. The arc length is calculated by taking R1 from (53, 19.8) until (60, 21.5) as part 1 in region 1.

Focus on quadratic line

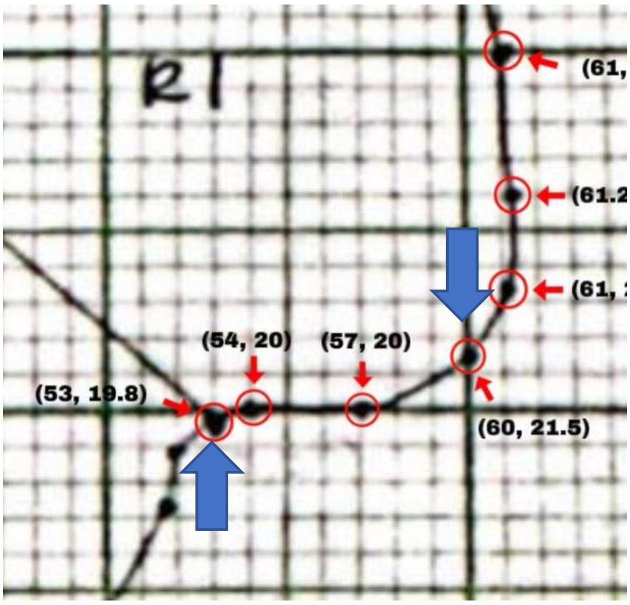
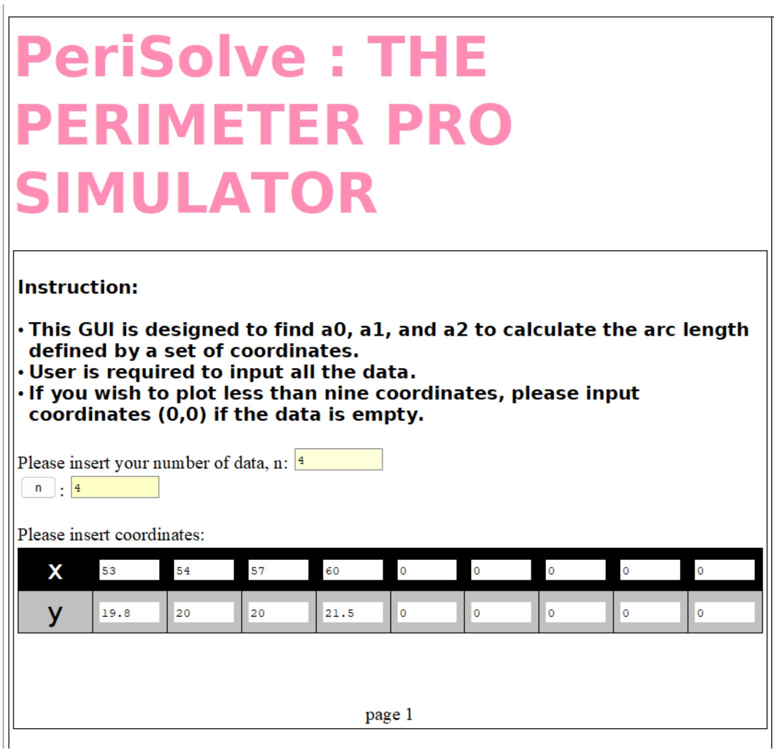


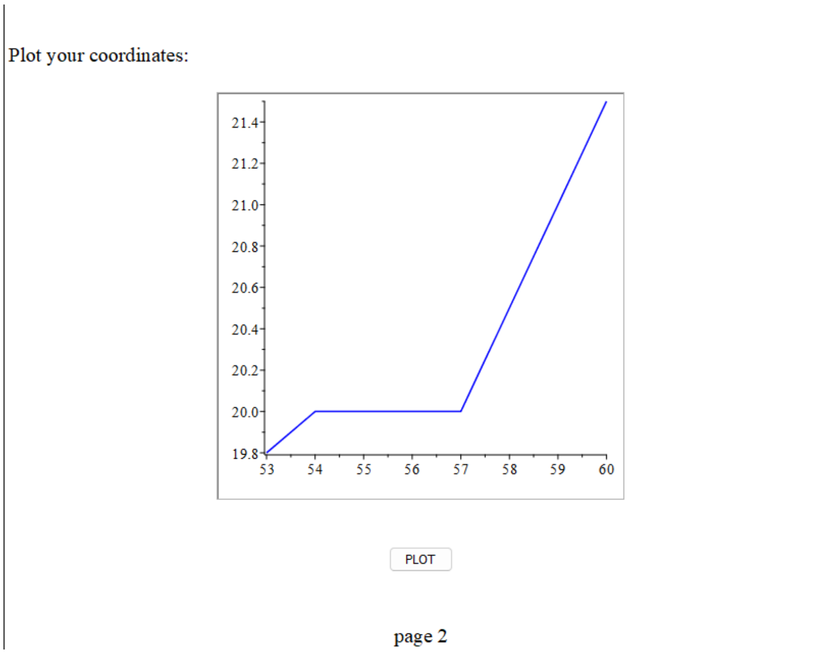
Figure 3: First part in Region 1 (quadratic line)

3.1 Solution

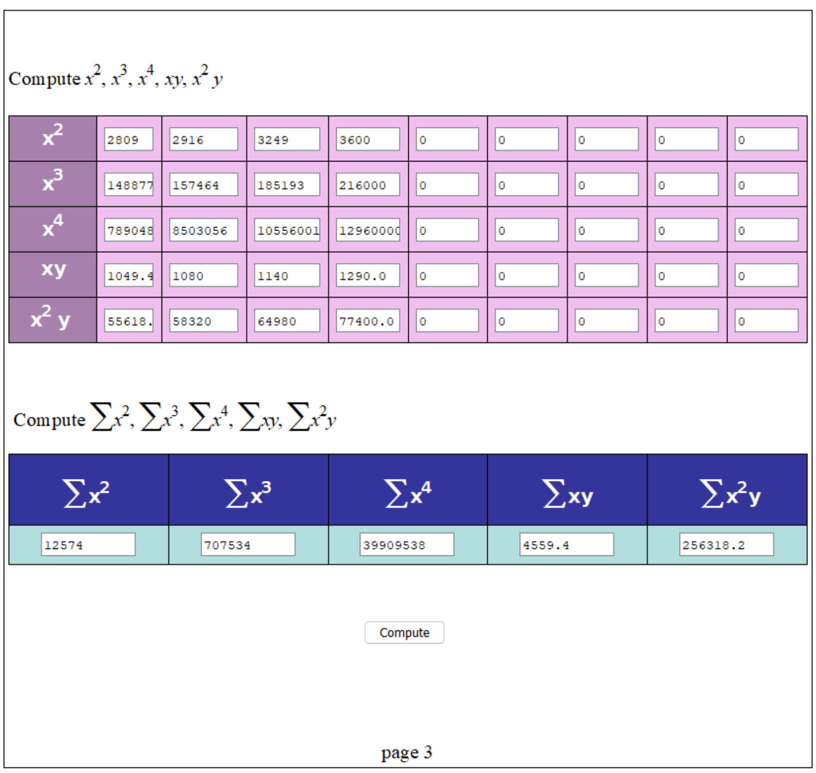
Step 1: Users identify their data and designate the number of sub-intervals, n which is 4. The coordinates are limited to just nine. Then, users must enter all the x and y coordinates. If the coordinates are below nine points, then just input 0 as the coordinates.



Step 2: When users select the 'Plot' button, a graph of the coordinates will show in second interface.



Step 3: When users click the 'Compute' button, the data x^2, x^3, x^3, x^4, xy *and* x^2y and the submission of the data will be shown in third interface.


Compute x^2, x^3, x^4, xy, x^2y

x^2	2809	2916	3249	3600	0	0	0	0	0
x^3	148877	157464	185193	216000	0	0	0	0	0
x^4	789048	8503056	10556001	12960000	0	0	0	0	0
xy	1049.4	1080	1140	1290.0	0	0	0	0	0
x^2y	55618.	58320	64980	77400.0	0	0	0	0	0

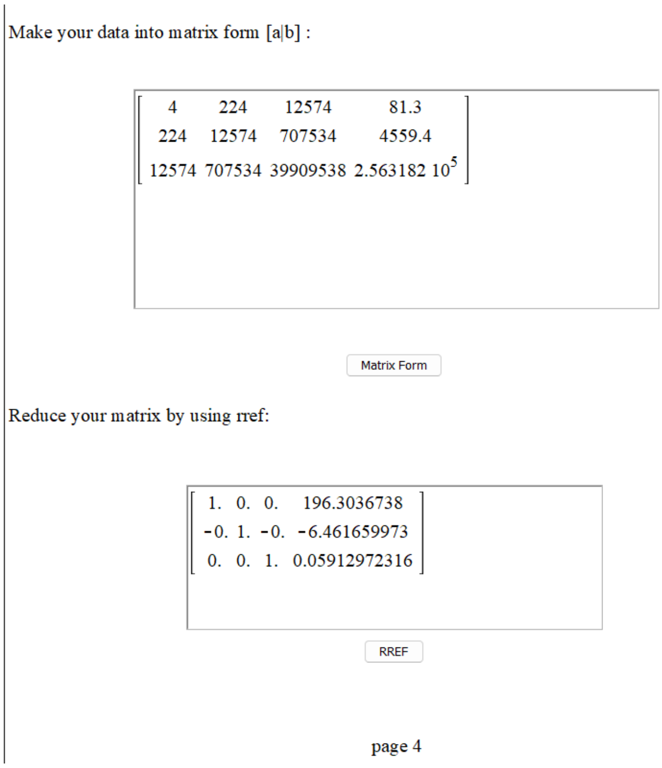
Compute $\sum x^2, \sum x^3, \sum x^4, \sum xy, \sum x^2y$

$\sum x^2$	$\sum x^3$	$\sum x^4$	$\sum xy$	$\sum x^2y$
12574	707534	39909538	4559.4	256318.2

Compute

page 3

Step 4: The 'Matrix Form' button will generate data in matrix form. When users select the RREF button, the data will be reduced using the reduced row echelon form (rref), as shown in fourth interface.

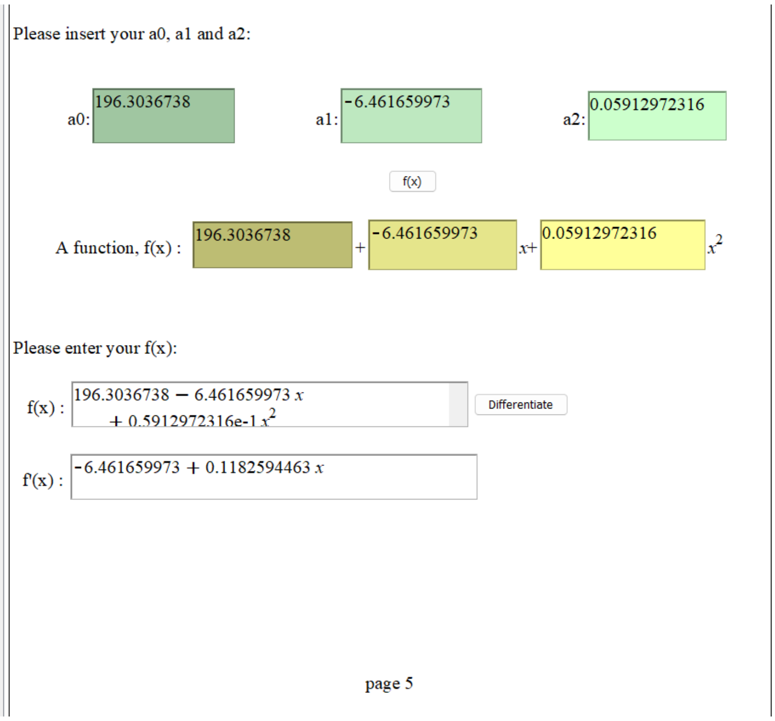


Step 5: After users press the 'RREF' button, the values a0, a1, and a2 will be displayed in fifth interface. To create a quadratic function,

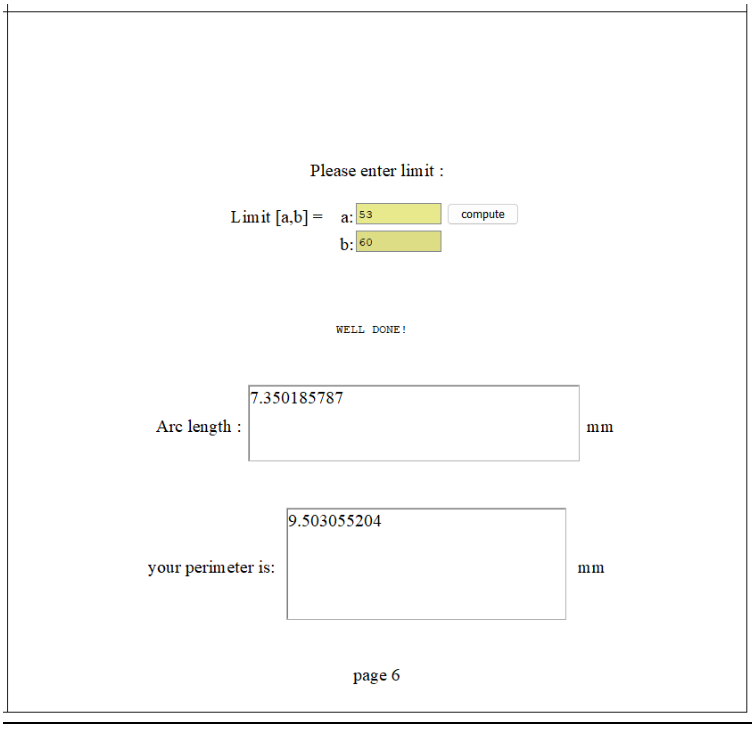
$$f(x) = a_0 + a_1x + a_2x^2 \tag{1}$$

users must click the 'f(x)' button. After receiving the function, users must enter it again and click the 'Differentiate' button to differentiate the function. The value of a0, a1 and a2 obtained in step 5 will be substituted into equation (1) to obtain equation (2).

$$f(x) = 196.1800 - 6.4573x + 0.05909x^2 \tag{2}$$



Step 6: Requires users to enter limits a and b. When users click the 'Compute' button, the arc length and true perimeter will appear, as shown in sixth interface.



In sixth interface, the system will solve for arc length by using this formula to obtain the actual perimeter. The formula for arc length given as in equation (1).

$$Arc\ length = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx \qquad (1)$$

$$Arc\ length = \int_{53}^{60} \sqrt{1 + [-6.4573 + 0.11818x]^2}dx = 7.3498 \text{ mm}$$

Since the ultrasound shows that 90.5 mm while in the plotted graph is 70 mm, hence

1 mm (in graph) = 1.2929 mm (actual). So, it can be concluded that the total arc length is 9.5026 mm.

Hence, the same steps were repeated for another region.

3.2 Result

As a result, the manual calculation of the arc length for part 1 in region 1 is 9.5026 mm, while using PeriSolve gives a result of 9.5031 mm, showing a small difference likely due to the precision of decimal places. PeriSolve may be using more decimal places internally, resulting in a slightly more precise calculation. Since there are many parts that need to be calculated for the arc length, it becomes difficult to do so manually. Therefore, PeriSolve is the best way to determine the arc length easily and quickly. By using PeriSolve all parts of each region can be determined as follows.

Table 1. Table for region 1 (R1):

Part	Perimeter
1	9.503055204
2	17.69744774

Total Perimeter of region 1: 27.2005 mm

Table 2. Table for region 2 (R2):

Part	Perimeter
1	9.527894973
2	17.07219776
3	10.43924768
4	14.49931078
5	22.15075223

Total Perimeter of region 2: 73.6894 mm

Table 3. Table for region 3 (R3):

Part	Perimeter
1	12.71720500
2	10.83508749
3	9.792233224
4	13.32778752
5	8.123850394

Total Perimeter of region 3: 54.7963 mm

Table 4. Table for region 4 (R4):

Part	Perimeter
1	3.485920437
2	34.56794922
3	6.176096246
4	3.976893813
5	2.450246634
6	7.953251098
7	3.481239790
8	11.78229894
9	8.162085781

10	2.734981769
11	15.66029799
12	5.120022838
13	8.173309341
14	6.493234285
15	4.074087185
16	9.232969847
17	4.214401912
18	7.619196357
19	8.452575804

Total Perimeter of region 4: 153.8110 mm

The total perimeter of a tumour can be determined by adding perimeter for region 1, region 2, region 3 and region 4 as follow.

$$Total\ perimeter = Perimeter\ of\ Region\ 1 + Perimeter\ of\ Region\ 2\ + Perimeter\ of\ Region\ 3 + Perimeter\ of\ Region\ 4$$

$$= 27.2005\ mm + 73.6894\ mm + 54.7963\ mm + \ 153.8110\ mm$$

$$= 309.4972\ mm$$

The tumour was measured on the plotted graph using a thread, yielding a measurement of 462 mm. A small difference between these two methods is observed due to an error. Next, researchers will calculate the percent error of the obtained perimeter using the percent error formula as below:

$$Error\ = \left|\frac{Exact\ perimeter - \ Approximated\ perimeter}{Exact\ perimeter}\right| \times 100\%$$

$$Error\ = \left|\frac{309.4972\ mm - \ 462\ \ mm}{309.4972\ mm}\right| \times 100\% = 49.27\%$$

4. DISCUSSION

The perimeter of the gastrointestinal stromal tumour (GIST) has been successfully calculated manually. However, it is time-consuming to calculate them one by one. To solve this issue, a Graphical User Interface (GUI) has been developed. PeriSolve is a graphical user interface (GUI) and professional simulator designed to simplify perimeter calculations. The uneven surface of the selected object can be measured easily with a mathematical formula. The GUI is expected to be very useful for everyone since it is a system. All users need to do is plot the coordinates of a selected object so that the system will be able to measure the perimeter. Calculating the object's perimeter can be useful in everyday life since it assists potential users of this GUI system in making decisions.

5. CONCLUSION

In conclusion, this study utilized the least squares method and the Gauss-Jordan elimination method to calculate the perimeter of a gastrointestinal stromal tumour (GIST), yielding a total perimeter of 309.4972 mm. These methods were found to be effective for this purpose, showcasing their broader utility in solving diverse problems. The least squares method, particularly in interpolation, aids in determining optimal lines, curves, and their arc lengths. This research serves as a comprehensive resource for understanding and implementing the least squares method, offering valuable insights for researchers and practitioners on its applications and implications across various fields.

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